GLIMPES OF GEOMETRY, 29-30 SEPTEMBER, PISA
ABSTRACTS

A state affair. Disputes and controversies among geometers around the 1901 Royal Prize, Umberto Bottazzini

The history of the 1901 Royal Prize seems particularly suitable for offering glimpses of algebraic and differential geometry in Italy at the turn of the twentieth century. As I will show in the talk, the contest turned into a true 'state affair' as regards Enriques and Castelnuovo's joint application to the Prize. At the same time the jury's judgment offered an interesting picture of the view then prevalent in differential geometry.

After Veronese and Cremona: the Cremona – Richmond configuration via Segre, Aldo Brigaglia

In 1877 Cremona published a paper [1] in which he re-interpretated the work of the young Giuseppe Veronese ([2]) in the geometry of cubic surfaces. He went through different stages, of growing abstraction. In the first stage he showed that projecting a nodal cubic surface from its singular point, O, we obtain exactly the theorems proved by Veronese about Pascal's Mysticum Hexagram. This construction is based on the known property that, if we do not consider lines and tritangent planes passing through O, there are exactly 15 planes concurrent by threes on 15 lines that lie, by threes, on the planes.
I will describe Cremona's extensions and abstractions comparing his work with Richmond's paper [3] which is always cited referring to Cremona – Richmond configuration and translates Cremona's approach in an analytical and hyperspatial one, using also Segre's results ([4]) about his famous "Segre primal". My main references are [5,6]

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[2] G. Veronese, Nuovi Teoremi sull'Hexagrammum Mysticum, *Memorie della R. Accademia dei Lincei,* (3), 1, 1876 – 77, pp. 649 – 703.
[3] H. W. Richmond, The Figure formed from six points in the space of four dimensions, *Quarterly Journal of Mathematics,* 31, 1900, pp. 125 – 160.
[5] H. Coxeter, Self – dual configurations and regular maps, *Bulletin of the AMS,* 56, pp. 413 – 455.
[6] I. Dolgachev, Abstract configurations in Algebraic Geometry, *Proceedings of the Fano Conference*, 2002.

Some Italian algebraic geometers and abstract configurations, Cinzia Cerroni

Theodor Reye in 1876 was the first to identify various geometrical configurations as illustrations of a general concept. The 1880's thus witnessed the first systematic studies of geometric configurations in the plane or space with various new discoveries by S. Kantor, V. Martinetti, A. Schönflies, C. Segre, G. Veronese, E. Steinitz

Many of the early investigations up until 1887, including those by S. Kantor, E. Steinitz and others, focused on those configurations that can be embedded in the real projective plane, thus the primary context of interest was geometry, just as for Reye. A new orientation toward the study of configurations arose, however, in the work of Martinetti, Schönflies, and E. H. Moore. In 1887, Martinetti developed a recursive method for constructing all configurations n3, and he applied this to construct all 31 configurations of type 113.

It is interesting that there do exist a few letters that S. Kantor wrote to Luigi Cremona after returning from Rome. One of these letters written on 25 December 1882, is of interest for this study. In this letter, Kantor mentioned that he attended Cremona's lecture courses for two semesters and summarized his recent work on configurations.

We investigate on the works of some Italian algebraic geometers about abstract configuration and about their influence on this theory. In particular we will analyse the works of Vittorio Martinetti. He was born in Mantova in 1859 and died in Milano in 1936. He studied in Pavia where he graduated in 1882. Since 1886 he taught in Messina where he remained with the exception of five years spent at the University of Palermo (1909-14) after the earthquake of Messina. He was also rector of the University of Messina (1900-1908).

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In the context of the Cremona school, some interventions by Ettore Caporali are worthy of note, which have remained unpublished and published posthumously.

An overview on projective differential geometry in Italy around 1900, Ciro Ciliberto

This talk is dedicated to an overview on the history of projective differential geometry, that is the study of properties of differential, or analytic, subvarieties of the real or complex *n* dimensional projective space, which are invariant under the group of projective transformations. I will especially focus on the Italian contributions to this discipline, starting from the end of XIX century till the first half of XX century. The main characters are C. Segre, G. Fubini, A. Terracini, E. Bompiani, and around them several other Italian scholars contributed to the development of the discipline. I will discuss the following issue: Can we speak of an Italian school of PDG?

Deformation problems for hypersurfaces around 1900: the contributions of Ricci-Curbastro, Bianchi and Bompiani, Alberto Cogliati

Soon after the publication of Riemann's *Habilitationsvortrag* and the introduction of *n*-dimensional metric geometry, a group of mathematicians including Richard Beez, Rudolph Lipschitz, Wilhelm Killing, Aurel Voss and Friedrich Schur began exploring the possibility of elaborating a general theory of hypersurfaces immersed in an *n*-dimensional Euclidean space. They easily realized that these new entities exhibit a number of unexpected and counterintuitive geometrical properties, such as their *general* rigidity: if $n \geq 4$, then the giving of a metric is sufficient to uniquely determine a hypersurface (except for its position in space).
This theorem, first obtained by Beez (1876) and later confirmed by Ricci-Curbastro (1883), was soon discovered to admit exceptions, thus opening the way to the search for a characterization of those hypersurfaces that are (genuinely) deformable.
Inspired by investigations of Schur, Bianchi (1905) and Umberto Sbrana (1909), a student of Bianchi at the Scuola Normale, gave a general solution to this problem. Finally, in 1914 Bompiani reexamined the work of Bianchi and Sbrana, by reinterpreting their solution in light of notions and techniques that he derived from projective differential geometry.
In this talk I will describe the historical process that led to Bianchi and Sbrana's classification result, by focusing mainly on Bianchi's work and its relationship with the investigations of Ricci-Curbastro and Bompiani.

Foreign inspiration and domestic tradition: the Göttingen-speaking mathematicians in Turin, Erika Luciano

The two courses in Higher Analysis and Higher Geometry delivered by Guido Fubini and Corrado Segre in 1916 and devoted, respectively, to number theory and elementary mathematics from an advanced standpoint account for an overall historical phenomenon which affected the mathematical Institute of Turin from the late 19th century to the mid-Twenties: the attempt to shape the organization of mathematical activities in Turin on the Göttingen model. In this paper, basing on correspondence and archive sources, we will outline how a community linked to a specific place (namely the Turin geometers Corrado Segre, Gino Fano, Guido Fubini and their protégés), organized the research activity, higher teaching, teachers training and library, in a given year (1916), trying to recreate in small scale the mathematical life lived at the Göttingen Mathematisches Institut in the era of Felix Klein.

Entanglements between the theory of equations, elliptic functions and algebraic geometry in Italy, in the second half of 19th century, Enrico Rogora

After a quick review of the contributions of Betti and Brioschi to the theory of algebraic equations, I shall discuss the solution of the quintic by means of elliptic functions from a geometrical point of view, following Gordan, Klein and Clebsch. By considering the way in which the theory of algebraic equations was taught in various university courses in Italy, I will make some remarks about "algebra in geometric fashion", its beauty and its limits.

Eugenio Bertini: teacher and researcher during his stay in Pavia, Riccardo Rosso

In this talk I will analyse unpublished lecture notes in Higher Geometry that Eugenio Bertini delivered in Pavia during the Academic year 1881-82. By comparing and contrasting their content with his lithographed notes and monographies that were published two decades later, we can have an idea of the evolution of his own research as well as of the change in the scientific background, in a period where the Italian school of geometry achieved remarkable results, thanks to the work of Castelnuovo, Segre, Veronese, just to mention those leading scientists that influenced Bertini's activity.

How absolute was the absolute differential calculus?, Tilmann Sauer

In 1900, Gregorio Ricci and Tullio Levi-Civita presented what they announced as a new tool for mathematical analysis. They referred to it as the ``absolute differential calculus". Its method was explicitly intended as a universally applicable calculus for problems in all areas of mathematical physics and beyond. However, its later reception was dominated by the fact that Marcel Grossmann in 1913 transformed its basic tenets into a tensor calculus that was taken up by Albert Einstein and used as the underlying mathematical framework for the general theory of relativity. In my talk, I will go back to the original formulation of the absolute differential calculus and reexamine its original intent and pecularities.

Levi-Civita's treatises on the absolute differential calculus, 1925-1928 -- research, teaching, dissemination, Rossana Tazzioli

The treatises on the absolute differential calculus collect the lectures given by Levi-Civita at the University of Rome from 1920/21 onwards. After the Italian edition (Stock, 1925), Levi-Civita published an English edition (Blackie, 1927) and then a German one (Springer, 1928). Through an analysis of the different editions, I aim to illustrate how Levi-Civita reinterpreted and rewrote some concepts and results already introduced in his previous papers -- in the joint memoir with Ricci Curbastro of 1901 or in the article on parallel transport of 1917 -- to adapt them to a student audience. His treatises had also a relevant influence on his research on differential geometry and general relativity, as for example in introducing the concept of "geodesic deviation" (today "Jacobi field"). Finally, his treatises were an important tool for promoting the Italian tradition in differential geometry: Levi-Civita not only referred to the works of Beltrami, Bianchi and Ricci Curbastro, but he often cited the articles of his own students and collaborators.